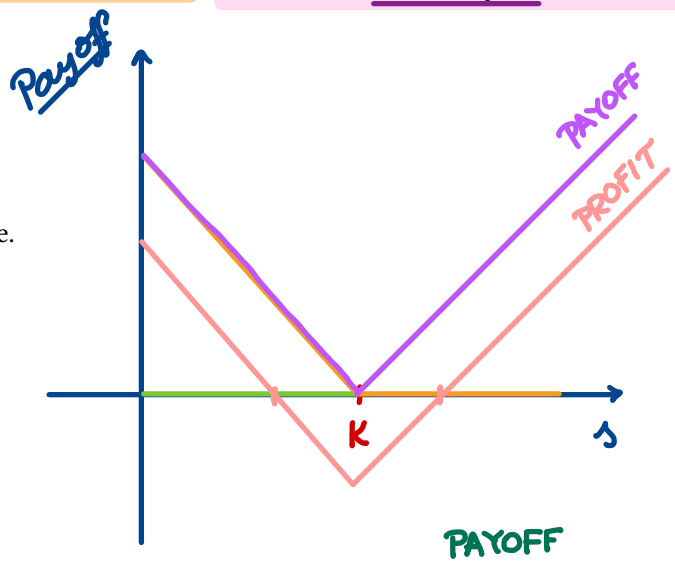
**Problem 12.3.** Consider a non-dividend paying stock whose current price is $95 per share. You model the evolution of this stock price over the following year using a one-period binomial tree under the assumption that the stock price can be either $120, or $75 in one year.

The continuously compounded, risk-free rate is 0.06.

A **straddle** consists of a long call and a long otherwise identical put. Consider a $100-strike, one-year European straddle on the above stock. What is the straddle's price consistent with the above stock-price model?

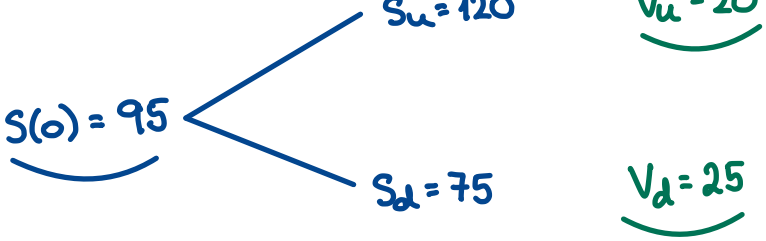
a. About $10
b. About $10.83
c. About $15.45
d. About $20.84
e. None of the above.




Payoff f'tion of the straddle

$v(s) = |s-K|$

In this problem $K=100$




$$\Delta = \frac{V_u - V_d}{S_u - S_d} = \frac{20-25}{120-75} = -\frac{5}{45} = -\frac{1}{9}$$

$$B = e^{-rh}\,\frac{u\cdot V_d - d\cdot V_u}{u-d} = e^{-0.06}\cdot \frac{\frac{120}{95}\cdot 25 - \frac{75}{95}\cdot 20}{\frac{120}{95}-\frac{75}{95}}$$

$$B = e^{-0.06}\cdot\frac{120\cdot 25 - 75\cdot 20}{45} = 31.392$$

$$V(0) = \Delta\cdot S(0) + B = -\frac{1}{9}\cdot 95 + 31.392 = 20.84$$

□

# Risk-Neutral Probability

Start w/ $V(0) = \Delta \cdot S(0) + B$

$$V(0) = \frac{V_u - V_d}{S_u - S_d} \cdot S(0) + e^{-rh} \cdot \frac{u \cdot V_d - d V_u}{u - d}$$

$S(0)(u-d)$

$$V(0) = \frac{1}{u-d}\left[(V_u - V_d) + e^{-rh}(u \cdot V_d - d V_u)\right]$$

$$V(0) = e^{-rh} \cdot \frac{1}{u-d}\left[e^{rh} \cdot V_u - e^{rh} \cdot V_d + u \cdot V_d - d \cdot V_u\right]$$

$$V(0) = e^{-rh} \cdot \frac{1}{u-d}\left[V_u(e^{rh} - d) + V_d(u - e^{rh})\right]$$

$$V(0) = e^{-rh}\left[V_u \cdot \frac{e^{rh} - d}{u-d} + V_d \cdot \frac{u - e^{rh}}{u-d}\right]$$

Add up to 1!

Both positive! (due to the no-arbitrage condition)

We choose to call the two fractions probabilities!

We define the risk-neutral probability of the stock price going up in a single period as

$$p^* := \frac{e^{rh} - d}{u-d}$$

$$V(0) = e^{-rT}\left[p^* \cdot V_u + (1-p^*) \cdot V_d\right]$$

$e^{-rT}$: discounting

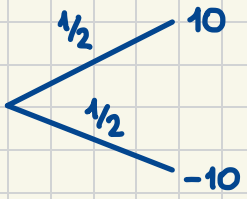
$\left[p^* \cdot V_u + (1-p^*) \cdot V_d\right]$: expected payoff under the risk-neutral probability

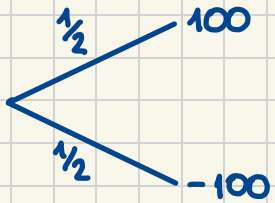
**Generalization:**

$$V(0) = e^{-rT} \cdot \mathbb{E}^*[V(T)]$$

Q: Why "risk-neutral"?

Imagine bets





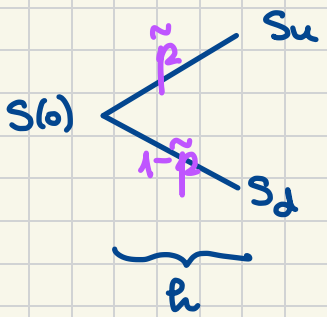

Consider a risk-neutral investor, i.e. one who is indifferent to risk and cares only about the expected wealth.

Q: What is the probability $\tilde{p}$ such that for a specific stock price tree this investor is indifferent between investing in the stock or in savings account?

→: Start w/ $S(0)$.

If we invest @ the ccrfir $r$, then our balance @ time-$h$ is $S(0)e^{rh}$

If we invest in the stock:




$$\mathbb{E}[\text{wealth}] = \mathbb{E}[S(h)]$$
$$= \tilde{p} \cdot S_u + (1-\tilde{p}) \cdot S_d$$
$$= \tilde{p} \cdot u \cdot S(0) + (1-\tilde{p}) \cdot d \cdot S(0)$$
$$= \left(\tilde{p} \cdot u + (1-\tilde{p}) \cdot d\right) \cdot S(0) = e^{rh}$$

$$\tilde{p} \cdot u + (1-\tilde{p}) \cdot d = e^{rh}$$
$$\tilde{p}(u-d) = e^{rh} - d$$

$$\boxed{\tilde{p} = \frac{e^{rh}-d}{u-d} = p^*}$$

□