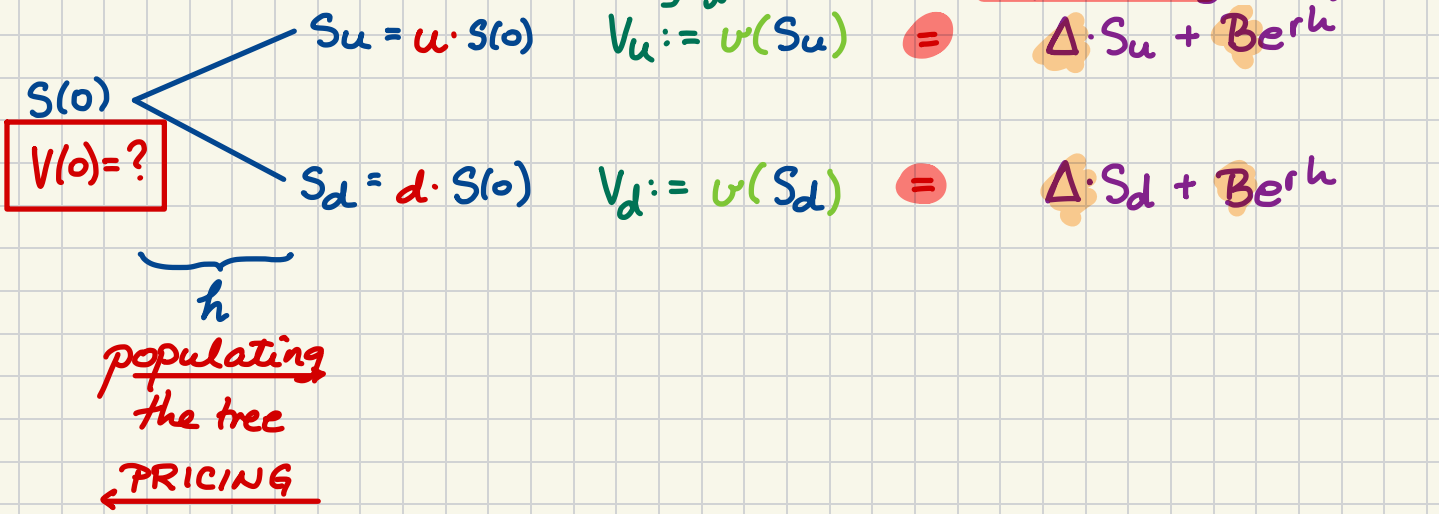
Binomial Option Pricing.



Goal: Pricing a European-style derivative security w/ exercise date @ the end of the tree, i.e., $T = h$

It's completely determined by its payoff f'tion: $v(\cdot)$

e.g., for a call: $v_c(s) = (s-K)_+$

for a put: $v_p(s) = (K-s)_+$

for a power option: $v(s) = (s^2-K)_+$

The payoff of our derivative security is the random variable

$$V(T) := v(S(T))$$

In the binomial model, any derivative security can be REPLICATED w/ a portfolio of this form:

@ time 0:

- Δ shares of stock
  - $\Delta > 0$ buying
  - $\Delta = 0$ "nothing"
  - $\Delta < 0$ short-selling

and

- B @ the ccrfir
  - $B > 0$ lending (buying a bond)
  - $B = 0$ "nothing"
  - $B < 0$ borrowing (issuing a bond)

If we can calculate Δ and B, then $V(0) = \Delta \cdot S(0) + B$

We get a system of two eq'ns w/ two unknowns:

$$\left.\begin{aligned} \Delta \cdot S_u + Be^{rh} &= V_u \\ -\Delta \cdot S_d - Be^{rh} &= -V_d \end{aligned}\right\} -$$

$$\Delta(S_u - S_d) = V_u - V_d$$

$$\boxed{\Delta = \frac{V_u - V_d}{S_u - S_d}}$$

unitless / shares of stock

$$Be^{rh} = V_u - \frac{V_u - V_d}{S_u - S_d} \cdot \underbrace{u \cdot S(0)}_{S_u}$$

$$S(0)(u-d)$$

$$Be^{rh} = \frac{u V_u - d \cdot V_u - u V_u + u \cdot V_d}{u-d}$$

$$\boxed{B = e^{-rh} \cdot \frac{u \cdot V_d - d \cdot V_u}{u-d}}$$

cash ($)

The University of Texas at Austin
In-Class Work 12

M339D Introduction to Financial Mathematics

October 16, 2026

Replicating Portfolios.

Problem 12.1. Let the CCRFIR be equal to 0.04. Consider a one-period binomial tree with every period of length one year used to model the stock price of a stock whose current price is \$80 per share. In the model, it is assumed that the stock price can either go up by \$5 or down by \$4.

You use the binomial tree to construct a replicating portfolio for a 78—strike call option on the above stock. What is the stock investment in the replicating portfolio?

→:

$$\Delta = \frac{V_u - V_d}{S_u - S_d}$$

PAYOFF

$S_u = 85$ $\quad V_u = (85-78)_+ = 7$

$S(0)$

$S_d = 76$ $\quad V_d = (76-78)_+ = 0$

$$\Delta = \frac{7}{85-76} = \frac{7}{9}$$

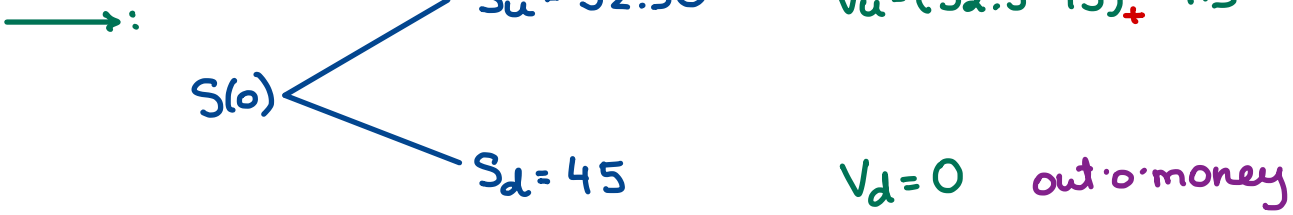
□

CCRFIR

Problem 12.2. Let the be equal to 0.04. Consider a one-period binomial tree with every period of length one year used to model the stock price of a non-dividend-paying stock whose current price is \$50 per share. In the model, it is assumed that the stock price can either go up by 5% or down by 10%.

You use the binomial tree to construct a replicating portfolio for a 45—strike call on the above stock. What is the risk-free investment in the replicating portfolio?

PAYOFF



$$B = e^{-rh} \cdot \frac{u \cdot V_d - d \cdot V_u}{u - d}$$

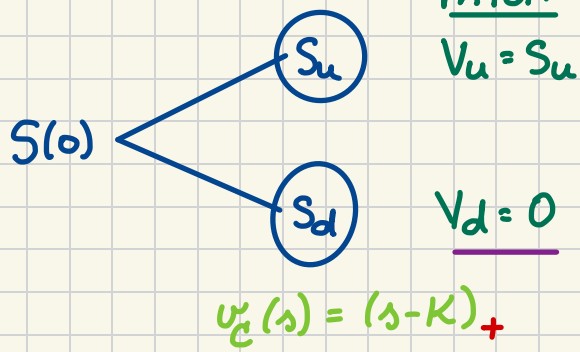
$$B = e^{-0.04} \cdot \frac{1.05 \cdot 0 - 0.9 \cdot 7.5}{1.05 - 0.90} = e^{-0.04} \cdot \frac{(-1)\,6.75}{0.15}$$

$$B = -43.24$$

□

Graphical Interpretation.

Consider a European call w/ exercise date @ the end of the tree and K such that $\boxed{S_d < K < S_u}$



In the replicating portfolio:

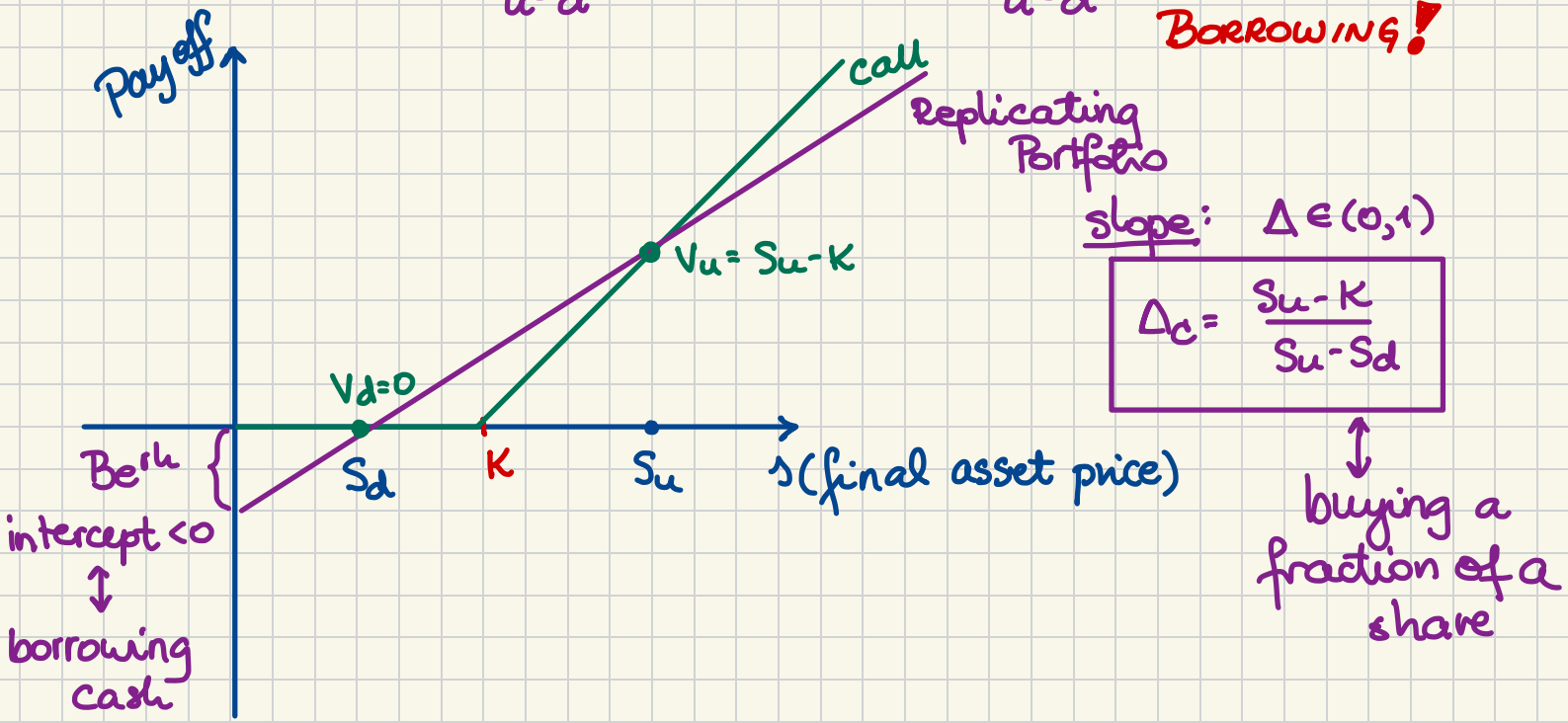
- $\Delta = \dfrac{V_u - V_d}{S_u - S_d} = \boxed{\dfrac{S_u - K}{S_u - S_d}} \in (0,1)$

Buy a fraction of a share of stock!

and

- $B = e^{-rh} \cdot \dfrac{u \cdot V_d - d V_u}{u-d} = -e^{-rh} \cdot \dfrac{d(S_u - K)}{u-d} < 0$

BORROWING!



slope: $\Delta \in (0,1)$

$\boxed{\Delta_C = \dfrac{S_u - K}{S_u - S_d}}$

buying a fraction of a share