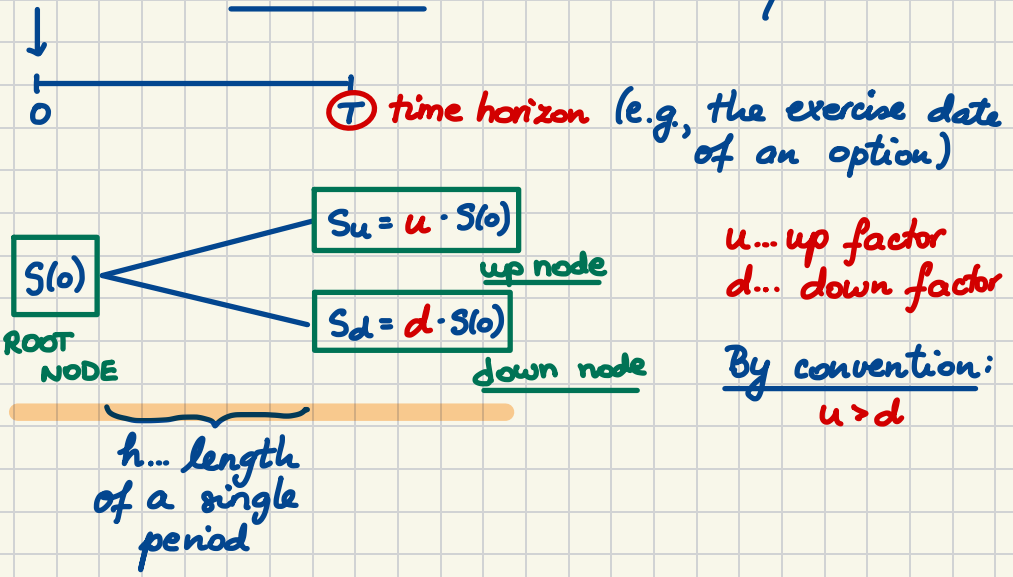
The Binomial Asset Pricing Model.

$S(0)$... the observeable initial asset price



u... up factor
d... down factor

By convention: $u > d$

one period: $S(T) = S(h)$ ... a random variable denoting the time-T stock price w/ two possible values S_u and S_d

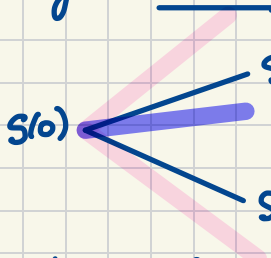
As a random variable:

- SIMPLE RETURN: $\frac{S(T)-S(0)}{S(0)}$
  - up node: $\frac{S_u - S(0)}{S(0)} = \frac{S_u}{S(0)} - 1 = u - 1$
  - down node: $\frac{S_d - S(0)}{S(0)} = \frac{S_d}{S(0)} - 1 = d - 1$
- CONTINUOUSLY COMPOUNDED: $\ln\left(\frac{S(T)}{S(0)}\right)$
  - up node: $\ln(u)$
  - down node: $\ln(d)$

Market Model.

- **riskless asset**: @ the ccr.fir r
- **risky asset**: non-dividend-paying stock

Imagine investing in one share of stock @ time 0:



WEALTH

S_u

S_d

At the risk-free interest rate, the amount $S(0)$ accumulates to $S(0)e^{rh}$ in the same time period.

No-Arbitrage Condition:

$$S_d < S(0)e^{rh} < S_u$$

$$d \cdot S(0) < S(0)e^{rh} < u \cdot S(0)$$

$$\boxed{d < e^{rh} < u}$$

Half-a-Proof.

Say, to the contrary, that $\boxed{e^{rh} \le d} < u$ ✓

Propose. Long the stock.

Verify. Profit = Payoff − $FV_{0,T}$(Initial Cost)

$= S(h) - S(0)e^{rh}$

down node: $S_d - S(0)e^{rh} = d \cdot S(0) - S(0)e^{rh}$

$= S(0)(d - e^{rh}) \ge 0$

up node: $S_u - S(0)e^{rh} = u \cdot S(0) - S(0)e^{rh}$

$= S(0)(u - e^{rh}) > 0$

Indeed, this is an arbitrage portfolio. □

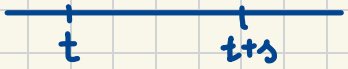
The other half-of-proof: $d < u \le e^{rh}$

⋮

Forward Binomial Tree.

"Def'n." The volatility σ is the standard deviation of realized returns on a continuously compounded scale and annualized.

Realized Returns.



Def'n. $R(t, t+s) := \ln\left(\frac{S(t+s)}{S(t)}\right)$

It satisfies: $\boxed{S(t+s) = S(t) \cdot e^{R(t,t+s)}}$