The University of Texas at Austin
Monte Carlo

M339D Intro to Financial Math

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The Strong Law of Large Numbers.

Let $\{X_k : k = 1, 2, ...\}$ be a sequence of **independent, identically distributed** random variables with a finite first moment

$$\mu_X := \mathbb{E}[X_1]$$

Then,

$$\frac{X_1 + X_2 + ... + X_n}{n} \overset{n\to\infty}{\longrightarrow} \mu_X$$

Moreover, let g be a function such that $g(X_1)$ is well defined. *What does this mean?*

Assume that $\mathbb{E}[g(X_1)]$ is finite.

Then,

$$\frac{g(X_1) + g(X_2) + ... + g(X_n)}{n} \overset{n\to\infty}{\longrightarrow} \mathbb{E}[g(X_1)]$$

Monte Carlo.

The idea of Monte Carlo is to "marry" our ability to simulate pseudo-random numbers using software with the *Strong Law of Large Numbers*.

Recipe.

Here is the algorithm for estimating "complicated" expectations.

- Draw simulated values from a specific distribution.
- Apply the function to the simulated values.
- Calculate the arithmetic average of the obtained quantities.

The result is a value **close** to the theoretical expectation.

Precision.

Notice that our estimates are merely simulated realizations of a random variable (the **sample mean** in the language of statistics). The measure of how **close** we expect our estimate to be to the theoretical mean is its **variance** or its **standard deviation**. We have

$$\begin{aligned}
\operatorname{Var}\left[\frac{X_1 + X_2 + ... + X_n}{n}\right] &= \frac{1}{n^2}\operatorname{Var}[X_1 + X_2 + ... + X_n] \quad \text{(independence)} \\
&= \frac{1}{n^2}(\operatorname{Var}[X_1] + \operatorname{Var}[X_2] + ... + \operatorname{Var}[X_n]) \quad \text{(identical distribution)} \\
&= \frac{1}{n^2}(n \operatorname{Var}[X_1]) = \frac{\operatorname{Var}[X_1]}{n}\,.
\end{aligned}$$

Ergo,

$$SD\left[\frac{X_1 + X_2 + ... + X_n}{n}\right] = \frac{SD[X_1]}{\sqrt{n}}\,.$$

In words: to increase our precision by a factor of η, we must increase our number of variates (draws) by a factor of η^2.