The University of Texas at Austin
Homework Assignment 2

Introduction to Financial Mathematics

September 07, 2026

Instructions: Provide your complete solution to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Prerequisite material.

Calculus.

Problem 2.1: *(5 points) Let $x > 0$. Then, we always have* $e^x > 1 + x$. True or false? Why?

Problem 2.2: *(5 points) Let $x > 0$. Then, we always have* $\ln(1 + x) < x$. True or false? Why?

Probability.

Problem 2.3: *(5 points) A coin for which* Heads *is twice as likely as* Tails *is tossed twice, independently. If the outcomes are two consecutive* Heads, *Bertie gets a \$15 reward; if the outcomes are different, Bertie gets a \$10 reward; if the outcomes are two consecutive* Tails, *Bertie has to pay \$5. What is the expected value of Bertie's winnings?*

Problem 2.4: *(5 points) At a tavern, Laszlo plays a dice game with a rigged six-sided die that shows an* even *number with probability $3/5$ and an* odd *number with probability $2/5$. He starts with a \$5 wager, rolls the die twice, independently, and his winnings **double** every time he rolls* even *(an* odd *roll leaves his current winnings unchanged). What is the expected value of Laszlo's winnings?*

Problem 2.5: *(5 points) At a boardwalk carnival, Bruno bets on a spinning wheel. The wheel's arrow lands at a uniformly random point $X \sim \text{Uniform}(0, 10)$ (measured in feet along the wheel's rim). His winnings, in dollars, are given by the following piecewise linear payoff*

$$W(x) = \begin{cases} -5 & \text{if } 0 \le x < 3 \\ 4x - 12 & \text{if } 3 \le x < 8 \\ 20 & \text{if } 8 \le x \le 10 \end{cases}$$

What is the expected value of Bruno's winnings?

Theory of Interest.

Problem 2.6: *(5 points) Roger initially deposits $4,000 in an investment fund which pays him $2,000 at time 1 and $4,000 at time 2.*

Sally gets $2,000 at time 0 and $4,000 at time 1, and deposits $5,460 at time 2 in return.

Both investments are governed by compound interest with the same annual ***effective*** *interest rate i and they have the same net present values.*

Find i.

a. About 9%
b. About 10.0%
c. About 11.5%
d. About 12%
e. None of the above

Problem 2.7: *(5 points) An investment grows by a factor of 1.05 during year 1 and by a factor of 1.08 during year 2. Let $r_1 = \ln(1.05)$ and $r_2 = \ln(1.08)$ denote the continuously compounded, risk-free interest rates earned during years 1 and 2, respectively.*

Let the ***total*** *continuously compounded, risk-free rate earned over the full two-year period be defined by*

$$r = \ln\left(\frac{a(2)}{a(0)}\right),$$

where $\frac{a(2)}{a(0)}$ is the overall accumulation factor for the two years combined.

Which of the following correctly gives r in terms of r_1 and r_2?

a. $r = r_1 + r_2$
b. $r = r_1 \cdot r_2$
c. $r = r_1 - r_2$
d. $r = \frac{r_1+r_2}{2}$
e. $r = e^{r_1} + e^{r_2}$

Problem 2.8: *(15 points) A nominal annual interest rate of j is compounded m times per year.*

i. *(5 points) What is the expression for the accumulated value $a_{m(t)}$ of $\$1$ after t years in terms of j, t and m?*
ii. *(5 points) Find $\lim_{m\to\infty} a_m(t)$.*
iii. *(5 points) Recall that the continuously compounded risk-free interest rate r is defined by*

$$r = \frac{a'(t)}{a(t)}$$

where $a(\cdot)$ stands for the accumulation function. What do the facts you proved above tell you about the interpretation of r?