The University of Texas at Austin
Homework Assignment 1

Introduction to Financial Mathematics

August 29, 2026

Instructions: Provide your complete solution to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Prerequisite material.

Pre-calculus

Problem 1.1. (5 points) Let the function $f : [0, \infty) \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} x - 300 & \text{for} \quad x \geq 300 \\ 0 & \text{otherwise} \end{cases}$$

Draw the graph of the function $g : [0, \infty) \to \mathbb{R}$ defined as

$$g(x) = f(x) - 50.$$

Problem 1.2. (5 points) Let the function $f : [0, \infty) \to \mathbb{R}$ be defined as

$$f(x) = x$$

Let the function $g : [0, \infty) \to \mathbb{R}$ be defined as

$$g(x) = \begin{cases} 0 & \text{for} \quad x < 500 \\ x - 500 & \text{for} \quad x \geq 500 \end{cases}$$

Draw the graph of the function $f - g$.

Problem 1.3. (5 points) We define the minimum of two real values in the usual way, i.e.,

$$\min(x, y) = \begin{cases} x & \text{if} \quad x \leq y \\ y & \text{if} \quad x \geq y \end{cases}$$

We define the maximum of two real values in the usual way, i.e.,

$$\max(x, y) = \begin{cases} x & \text{if} \quad x \geq y \\ y & \text{if} \quad x \leq y \end{cases}$$

Then, for every x and y we have that

$$x - \min(x - y, 0) = \max(x, y).$$

True or false? Why?

Problem 1.4. (5 points) We define the maximum of two real values in the usual way, i.e.,

$$\max(x,y) = \begin{cases} x & \text{if } \ x \geq y \\ y & \text{if } \ x \leq y \end{cases}$$

Then, for every x and y we have that

$$\max(x,y) = \max(x-y,0) + y.$$

True or false? Why?

Problem 1.5. (5 points) Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two functions given by

$$f(x) = |x - 10|$$

and

$$g(x) = \begin{cases} \min(x,4) & \text{if } \ x \geq 0 \\ 0 & \text{if } \ x < 0 \end{cases}$$

Then, $g(f(7))$ equals ...

a. 0
b. 3
c. 4
d. 7
e. None of the above

Probability.

Problem 1.6. (5 points) Let $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ be an outcome space, and let $\mathbb{P}$ be a probability distribution on Ω. Assume that $\mathbb{P}[\{\omega_1, \omega_2\}] = 1/3$, $\mathbb{P}[\{\omega_2, \omega_3\}] = 1/4$, and $\mathbb{P}[\{\omega_1, \omega_3\}] = 1/9$. Find $\mathbb{P}[\{\omega_4\}]$?

Problem 1.7. (10 points) Let $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$ be a probability space. We denote by p_k the probability of the elementary outcome ω_k, i.e., $p_k = \mathbb{P}[\{\omega_k\}]$ for $k = 1, ..., 5$. You are given that p_k / p_{k-1} is constant for $k = 2, 3, 4, 5$. You are also given that $p_1 = 16/31$. Find p_5.

a. 1/31
b. 2/31
c. 4/31
d. Not enough information is given.
e. None of the above.

Problem 1.8. (5 points) Emmanuel entered an extra special kind of game with his friend Fischer. First, they toss a fair coin. If the coin comes up heads, Emmanuel gives $\$5,000$ to Fischer. If the coin comes up tails, Fischer gives $\$2,000$ to Emmanuel. Then, regardless of the outcome of the first cointoss, they toss the same fair coin again. If it comes up heads, Emmanuel gives Fischer \$4,000. If the coin comes up tails, Fischer gives \$3,000 to Emmanuel. What is the expected cashflow, i.e., what is the expected amount of money that changes hands and who gives it to whom?

Problem 1.9. (5 points) Let Y be a random variable such that $\mathbb{P}[Y = 2] = 1/2$, $\mathbb{P}[Y = 3] = 1/3$ and $\mathbb{P}[Y = 6] = 1/6$. What is $\mathbb{E}[\min(Y, 5)]$?